Measurement of angular velocities using electrical impedance
tomography

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Abstract

Monitoring the dynamics within process equipment such as impeller-based mixers, hydrocyclones and centrifugal separators presents a challenge to instrument engineers because of the need to make real-time measurements non-intrusively. For such processes, the predominant component of velocity is angular in nature. In this paper a novel technique based on electrical impedance tomography (EIT) is introduced for measuring and mapping these angular velocities. The technique uses two conventional sets of electrode arrays on the same imaging plane, with one set displaced from the other by a known angle. The images generated from the two sets of electrodes are then cross-correlated with each other yielding time-shifted functions that are directly related to the angular velocities at discrete points on the imaging plane. Results from a lab-scale experimental measurement demonstrate the viability of the technique. © 1998 Published by Elsevier Science Ltd. All rights reserved.

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1. Introduction

The non-intrusive measurement of angular velocity profiles within process equipment is a challenging task that few instruments meet. The most common measurement technique, digital image velocimetry, is based on visual tracking of rotating objects [1–3]. This technique, which depends on optical sensing, is not suitable for making sub-surface velocity measurements of opaque fluids.

A relatively newer tomographic technique, electrical impedance tomography (EIT) the impedance field in, say, a process vessel are determined from measurements of potential differences between flush mounted electrodes along the boundary of the vessel created by the injection and withdrawal of a small amount of current from other electrodes. EIT has many advantages, such as low-cost, high-speed data-capture and imaging, adaptability and very few safety concerns. EIT imaging is also not limited by the opacity of fluids, such as concentrated suspensions.

EIT has been successfully employed in the study of the phase distribution of numerous processes such as hydrocyclones [5,6] and fluid–fluid mixing [7]. The study of mixing by Mann et al. [7] employed EIT for imaging the resistivity fields at different planes as the pulse of strong salt solution disperses through a less conducting fluid. The resulting EIT images clearly illustrated transients in the mixing, and they noted the need to determine velocity field associated with the mixing. While it could be possible to extend the work of Hayes [8] to estimate axial velocities by directly cross-correlating images at adjacent planes, a method for determining angular velocities by means of EIT remains to be developed.

In this paper the measurement of angular velocities of rotational motions using EIT is described. Two approaches to the problem are examined. The first cross correlates data obtained from two positions in the domain from images generated from a single EIT electrode set to determine the angular velocity profile. The second technique uses two sets of electrodes on the same plane with one set displaced from the other by a known...
Images generated from the two sets of electrodes are correlated with each other yielding time-shifted functions that are directly related to the angular velocities at discrete points on the plane. Both approaches allow the measurement of the angular velocity by tracking anomalies in the conductivity field.

The outline of the paper is as follows. First, an image reconstruction algorithm for EIT is reviewed. Next, the basic principles of velocity measurement using cross-correlation are briefly described. The two methods of velocity measurement are then presented and evaluated using synthetic measurements from computational simulations. Lastly, results from a laboratory experiment are reported and the paper concludes with a discussion of future prospects for this technique for measuring angular velocities.

2. EIT image reconstruction

In order to minimize the effects of electrode contact impedance, a four-electrode adjacent measurement strategy (Fig. 1) is commonly used for the acquisition of EIT data. In this strategy current $I$ is injected into one electrode of a selected pair of adjacent electrodes and collected from the second of the pair. The resultant voltage differences $V$ impressed on the remaining adjacent pairs of electrodes are then measured and logged. Next, another adjacent pair is selected for current drive, followed by voltage measurements. The process is repeated until all the adjacent electrode pairs have been current-driven.

Using the boundary measurements, an approximation to the resistivity distribution within the domain is calculated with a numerical algorithm. Many inverse strategies for EIT exist within the literature [9–11]. For this particular project, a one-step method for image reconstruction based on sensitivity weighted back projection [9] was used. The image domain is divided into discrete elements, and each element is assigned a value $P$, where

$$ P(x,y) = \frac{\sum_d \sum_m S_{d,m} \left( \frac{V - V'}{V} \right)_{d,m}}{\sum_d \sum_m S_{d,m}}. $$

For a domain of homogenous reference resistivity, $V_{d,m}$ is the voltage measured on electrode pair $m$ for current-drive pair $d$. For the same domain but with a perturbed resistivity distribution, the corresponding voltage measured is $V'_{d,m}$. $S_{d,m}$ is the sensitivity coefficient relating the measurement $d,m$ to the position $(x,y)$. The sensitivity coefficient for each pixel area $A$ within the domain can be evaluated as

$$ S_{d,m} = \iint_A \frac{\nabla \Phi_m}{I_m} \cdot \frac{\nabla \Phi_d}{I_d} \, dx \, dy. $$

Here, $\Phi_m$ is the potential field due to the current drive pair $m$ carrying a current $I_m$ for the case of the domain having a known homogenous resistivity. Likewise, for when the domain assumes a possibly different homogenous resistivity, $\Phi_d$ is the potential field due to the current-drive pair $d$ carrying a current $I_d$. The potential fields $\Phi_d$ and $\Phi_m$ can be readily computed analytically for certain simple geometries, or numerically in general.

For the purposes of this paper, knowledge of the relative values of the pixels is sufficient. Using a non-linear reconstruction method would provide more accuracy on the magnitude of the contrast, but it is really the spatial variation that is needed so the fast, linearized reconstruction is sufficient for the purposes here. If accurate information on concentration of the tracer is desired, then a non-linear reconstruction algorithm would be necessary. Note for small changes from the homogeneous resistivity, $P(x,y)$ is a measure of change in logarithmic resistivity [9] such that

$$ P(x,y) = \ln[p'(x,y)] - \ln[p]. $$

Here $p$ is the reference homogeneous resistivity and $p'(x,y)$ is the resistivity which differs by small perturbations from the uniform case.

3. Principle of velocity measurement

3.1. Basic approach

The measurement of the velocity of a flow process is usually accomplished by cross-correlating signals, such
as direct measurements of voltage or indirect measurements like pixel values, obtained at two separate locations in a flow segment. Fig. 2 illustrates the typical locations of sensors for measurements of axial and angular velocities.

If the signal at time $t$ obtained at positions $A$ and $B$ are $V_A(t)$ and $V_B(t)$ respectively, the cross-correlation function is:

$$R_{AB}(\tau) = \int_{-\infty}^{\infty} V_A(t) V_B(t + \tau) d\tau.$$  \hspace{1cm} (4)

The cross-correlation $R_{AB}(\tau)$ will have a maximum value at a time equal to the delay between the signals $V_A(t)$ and $V_B(t)$. For a time delay $\tau_1$, the axial velocity of the material in Fig. 2(a) is given by

$$v = d/\tau_1,$$  \hspace{1cm} (5)

where $d$ is the distance separating the sensors. For a rotating flow [Fig. 2(b)] with sensors separated by an angle $\theta$, the angular velocity is given by

$$\omega = \theta/\tau_1.$$  \hspace{1cm} (6)

For sampled data, the digital equivalent of Eq. (4) can be expressed as

$$R_{AB}[p] = \sum_{m=-\infty}^{\infty} V_A[m] V_B[m + p],$$  \hspace{1cm} (7)

where $m,p = \ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots$.

### 3.2. Measurement of angular velocity using one set of electrodes

In EIT image reconstruction it is common to have the cross-sectional plane of interest divided into finite elements. The resultant image is composed of these finite elements with each element having a value that is indicative of the resistivity of the region it occupies. The profile, of say the concentration of a suspension, can thus be obtained from the variations of the finite element values since particle concentration can be deduced from measurements of resistivity [12].

A direct approach for extracting angular velocity profiles from images generated by EIT is to cross-correlate values of adjacent finite elements for the particular case where the elements are arranged in rings as shown in Fig. 3(a). Assigning the angular velocity computed from the cross-correlation of two adjacent elements to one of the element gives

$$\omega_n = \frac{\theta_n - \theta_{n+1}}{\tau_n} (n = 1, 2, \ldots, N_R - 1)$$  \hspace{1cm} (8)

where $\omega_n$ is the angular velocity assigned to the region occupied by element number $n$, $\theta_n$ is the angular position of element $n$, $\tau_n$ is the signal delay computed from cross-correlation of elements $n$ and $n + 1$, and $N_R$ is the last element in the ring $R$. To obtain the angular velocity of the last element $N_R$ in a ring, the values of $N_R$ are cross-correlated with those of the first element in the ring.

In order to test the technique, the EIT potential fields for the rotational motion of a small-sized resistivity anomaly were computationally simulated using the finite-element method with the grid shown in Fig. 3(b). One of the 512 elements in the mesh was made five times more resistive, the boundary potentials were computed with the finite element method, and then the process was repeated as the region of higher resistivity rotated to the next element.

The velocity profiles determined from the simulated motions of an object rotating in a counter-clockwise direction at 0.09 and 0.18 rad/s are shown in Fig. 4. Sixteen electrodes were employed, with a total of 20 frames captured at 1 frame/s. Shown are the results of superpo-
position of measured velocities at the sampled instances of the 20 frames used for the cross-correlation, creating a trace of the path and velocity of the rotating element. The trajectories of the motions are correctly represented by the peak cross-correlation coefficients shown in Fig. 4(b) and Fig. 4(d). The peak cross-correlation coefficients were used for thresholding: elements with cross-correlation coefficients less than half of the maximum coefficient value were assigned zero velocity.

In Fig. 4, a total number of 128 finite elements were used in the image reconstruction prior to cross-correlation. The values estimated from the velocity profiles are fairly accurate. For the same simulated motions, the velocity profiles resulting from the use of a mesh of 3136 elements are shown in Fig. 5. The results clearly indicate inaccuracies, which arise from the use of a large number of finite elements (despite the fact that a larger number of finite elements tend to produce more accurate and sharper images).

This is because of the noise and spread inherent in EIT image reconstruction techniques, the consequence being that adjacent elements take on equal signals that result in zero time shifts when cross-correlated. To avoid this velocity imaging problem, one could use a grid no finer than the object being imaged, as evidenced from the comparison of Fig. 4 and Fig. 5. But then, the higher resolution of resistivity afforded by a finer grid is lost. Moreover, the proper grid scale necessary to resolve velocity may not be known a priori. That is, one does not know in advance the size of the anomaly being imaged. Thus, a more robust method is necessary, and such a method is described in the next section.

3.3. Measurement of angular velocity using two sets of electrodes

This method employs two sets of conventional electrodes for data capture. The disposition of the electrodes is illustrated in Fig. 6. Both sets of electrode are in the same plane, but one set is displaced from the other by a known angle \( \theta \). A complete scan, therefore, results in two images in essence viewed from two different angles. Measuring the angular velocity now requires cross-correlating the pixel strength of elements generated from the two sets of the electrodes. In this case the angular velocity for the individual elements is given by

\[
\omega_n = \theta / \tau_n,
\]

where \( \tau_n \) is the delay time obtained from the cross-correlation of the values of element \( n \) derived from the two sets of data. For the digitally sampled data the equation for the cross-correlation function of element \( n \) is

\[
R_{ABn}[p] = \sum_{m=0}^{T-1} V_{An}[m]V_{Bn}[m + p],
\]

where \( T \) is the total number of frames sampled, \( m \) and \( m + p \) are frame indices, \( n \) is the finite-element index, and \( V_{An}[m] \) and \( V_{Bn}[m] \) are the pixel value of the \( n \)th element in the \( m \)th sampled frame obtained from the \( A \) and \( B \) set of electrodes, respectively.

This method for measurement of angular velocities was tested on simulated motions of a rotating object. The
test object, whose area is 0.2% of the imaging plane, is rotated along a locus which is 3/4 radius measured from the center of the imaging plane. Using two sets of electrodes each composed of 16 electrodes, with one set displaced from the other by 11.25°, superimposed velocities for counter-clockwise motions at 0.09 and 0.18 rad/s are shown in Fig. 7. The velocities shown are the results of superposition of measured velocities at the sampled instances of the 20 frames used for the cross-correlation. The total number of finite elements used in the original image reconstruction was 3136. It should be mentioned that the associated cross-correlation coefficients similar to those shown in Fig. 4(b) and Fig. 4(d) were used for thresholding. Due to the interleaving nature of the two sets of electrodes, additional care was taken to correct for periodic sign reversal of the angular velocity using the following scheme:

\[
\text{If } [\varphi_n \mod (4\pi/E_T)] > (2\pi/E_T) \text{ then } \omega_n = -\omega_n.
\]

Here \(\omega_n\) is the angular velocity of element \(n\), \(\varphi_n\) is the positional angle of element \(n\), and \(E_T\) is the total number of electrodes used. The transition zones of velocity sign reversal are marked by periods of zero velocity. As can be observed, the resulting measured velocities accurately represent the simulated motions.

The maximum velocity that can be obtained from an evenly spaced array of electrodes is given by

\[
\omega_{\text{max}} = \theta / t_s = \frac{2\pi}{E_T \cdot t_s},
\]

(11)
where \( E_T \) is the total number of electrodes used and \( t_s \) is the sample interval between complete frames of data. Using the four-electrode adjacent measurement strategy described in Section 2, the number of independent measurements per set of electrodes is

\[
M = N \left( \frac{N - 3}{2} \right)
\]

(12)

where \( N = E_T/2 \) is the number of electrodes per set.

If each measurement takes time \( \tau \), then \( t_s \) is given by

\[
t_s = 2N \left( \frac{N - 3}{2} \right) \tau,
\]

(13)

and hence

\[
\omega_{\text{max}} = \frac{2\pi}{E_T \cdot t_s} = \frac{\pi}{N^2(N - 3)\tau}.
\]

(14)

Thus, in the limit of large number of electrodes, \( \omega_{\text{max}} \) is proportional to \( (N^0\tau)^{-1} \). So for a fixed \( \tau \), fewer evenly spaced electrodes must be used to image higher velocities, or the measurement rate must be increased. Subsample delay estimation [13,14] is an alternative method of measuring smaller delays, which can improve the resolutions of angular velocities, and possibly extend \( \omega_{\text{max}} \). This method commonly involves interpolating between sampled data in the vicinity of the extrema of the correlation function.

Using an \( 8 \times 2 \) electrode setup, the reconstructed angular velocity profiles of the aforementioned simulated motions are presented in Fig. 8. Fig. 8 presents the result for \(-0.36 \text{ rad/s}\) motion, a velocity which is twice as high as that permissible for the case of \( 16 \times 2 \) electrode setup, for the same sample interval \( t_s = 1\text{ s} \). The main advantage of the \( 16 \times 2 \) electrode setup is in the spatial resolution of the velocities. Its demerit hinges on limits of speed. The \( 8 \times 2 \) electrode setup can profile velocities twice as high as the \( 16 \times 2 \) electrode setup for the same frame capture-rate. Furthermore, only 40 measurements are required for a complete frame scan, whereas in the \( 16 \times 2 \) electrode setup 208 measurements are required. Thus from Eq. (14), it can be shown that
Fig. 7. Velocity profiles from images generated from $16 \times 2$ electrode measurements. The total number of elements used in the image reconstruction was 3136. The rotating object was positioned at $3/4$ of the radius from the center of the vessel. The superimposed velocity profiles are for (a) $-0.09$ rad/s and (b) $-0.18$ rad/s, each for a duration of 20 s.

Fig. 8. Velocity profiles from images generated from $8 \times 2$ electrode measurements. The total number of elements used in the image reconstruction was 3136. The rotating object was positioned at $3/4$ of the radius from the center of the vessel. The superimposed velocity profiles are for (a) $-0.09$ rad/s and (b) $-0.18$ rad/s and (c) $-0.36$ rad/s, each for a duration of 20 s.
the $8 \times 2$ electrode setup can image approximately 10 times the velocities achievable by the $16 \times 2$ electrode setup for a given measurement rate.

Simulated motions involving two test objects rotating in opposite directions were also investigated. One of the objects rotated at twice the speed of the other. Fig. 9 shows the reconstructed angular velocity profiles for both the $16 \times 2$ and $8 \times 2$ electrode setups. Counter clockwise rotation is indicated by a negative angular velocity, whereas a clockwise direction is represented by a positive angular velocity. Reconstruction noise (image smear) does however affect the accuracy particularly for the slower object. The accuracy can be improved by taking larger data sets, noting that the profiles in Fig. 9 were from 18 complete frame scans.

4. Experimental results

A lab-scale experiment was conducted to test the viability of using real EIT data for the measurement of angular velocities. The experimental setup (Fig. 10) was comprised of a cylindrical vessel with an inner diameter of 11.4 cm and a height of 25 cm. A circular section of the vessel (12 cm from the bottom) was fitted with 16 flush-mounted stainless-steel plate electrodes, each with 1 cm$^2$ contact area. For the purpose of the experiments, the electrodes were configured as two sets of eight interleaved electrodes. These electrodes were wired to an EIT system [15] that was programmed to make measurements using a 30 kHz current excitation signal of amplitude $\pm 11.531$ mA. The vessel was filled with a saline solution of conductivity 1.7 mS/cm. Afterwards, a glass rod (1 cm diameter) was dipped into the saline solution and rotated along the 3/4 radius of the vessel with the aid of an electric motor. Data was collected by the EIT system at a frame scan interval of 0.057 s.

The velocity measurement technique based on two interleaved electrode arrays (Section 3.3) was applied to measure the velocities of the rotating rod. The angular velocity profiles for three rotational speeds of $-1.35, -2.89,$ and $-6.83$ rad/s are presented in Fig. 11.

Fig. 12 shows the estimated velocities of non-zero pixel elements. Using these values, we were able to estimate the average measured angular velocities as $-2.36, -3.12,$ and $-6.52$ rad/s with corresponding errors of 74.8, 7.9 and 5.35%. The agreement is very good for the two faster velocities. It should also be mentioned that the online measurement data are not noise free, and this coupled with reconstruction noise and limited data set (14 frames in this case) account for these reasonably small errors. These errors can be significantly reduced by increasing the number of frames buffered for cross-correlation.

The excessive error in estimating the slowest rotation of $-1.35$ rad/s is due to the sampling window being too short to allow for a significant displacement of the glass rod which is a function of the limitations of electronics hardware used in the experiments rather than the algorithm. We are currently upgrading our EIT system so we can buffer an adequate number of frames for more accurate cross-correlation results. The EIT system upgrade will also encompass software modification leading to faster frame capture, and hence better velocity resolution.

![Fig. 9](image_url)

Fig. 9. Superimposed angular velocity profiles of two objects rotating in opposite directions, with one of the objects rotating at twice the speed of the other. The total number of elements used in the image reconstruction was 3136. The rotating objects were positioned at 3/4 of the radius from the center of the vessel; (a) is for simulated measurements using $8 \times 2$ electrode setup; (b) is for simulated measurements using $16 \times 2$ electrode setup. Each simulation was for a duration of 18 s.
5. Conclusions

In this paper we have presented two different approaches to imaging angular velocities in a given domain using EIT. In the first approach, a set of electrodes mounted at the boundary of the domain is used for data collection. The velocities at the image pixels were determined by cross-correlating the resistivities estimated at adjacent pixel locations. The accuracy of this approach is dependent on the size of the pixels in relation to the anomaly being imaged as demonstrated in Section 3.2. This is rarely known a priori when imaging. A second approach that frees one of this constraint is based on an EIT data collection strategy that uses two sets of electrodes with one set displaced from the other by a known angle. The angular velocities are determined by cross-correlating the sets of images derived from the two sets of electrodes. The second approach is more robust, and is not dependent on the size or arrangement of the pixels. Its demerit is that it takes twice as long to collect data compared to the first approach. Test results based on synthetic data from computational simulations are very accurate in the case of the second approach. The images show estimated velocity fields that trace the trajectories of rotating objects, while identifying the direction of rotation by associating an appropriate positive or negative sign to each velocity. Frames collected in a shortened sampling window, each window comprising of adequate number of frames for cross-correlation, can be used to obtain instantaneous velocities.

Laboratory experiments were conducted to evaluate the second method using real data. The resulting measurement of the velocities are very good (within 10%) for velocities sufficiently fast to show a change in the position in the anomaly within the amount of data buffered by the EIT hardware used in the experiments. The experiments also point out that the buffer size must be larger to measure slower velocities accurately, but this is not a limit on the proposed method for measurement of the angular velocities, but a limit on the current hardware in our laboratory. As described in Section 3.3, the resolution of the velocity image is directly related to the EIT frame sample interval. Furthermore, correlation functions are more accurate for a large number of samples. Thus, one can achieve higher accuracy of velocities using a rapid EIT system that is capable of high frame rates (say 10 times the Nyquist rates). The EIT system should also be able to buffer a large number of frames.

We have been able to show that EIT can be applied to imaging angular velocities in a given domain. It should be noted, however, that cross-correlation for estimating delays only works correctly where the signals involved vary with time. Thus, it is required that the EIT generated resistivity images be time variant, particularly along the radii. This calls for some time dependent perturbations in the domain, which we have represented by the use of test objects. The test objects for practical purpose need not be a solid material. It could be a fluid of a certain conductivity that is injected into a moving fluid of a different conductivity. Hence, EIT could be used to map the two fluid phases and estimate the angular velocities of the fluid interfaces. Mann et al. [7] studied mixing in 3-D by using EIT to monitor the dispersion of a strong salt solution in a less conductive fluid. Their results clearly demonstrated that EIT can detect mixing motions by a series of resistivity images that progressively changed at different planes. If our proposed angular
Fig. 11. Velocity profiles of a rotating rod experiment. The images were generated from an EIT system configured for $8 \times 2$ electrode measurements. Total number of elements used in the image reconstruction was 3136. The superimposed velocity profiles are for (a) $-1.35$ rad/s; (b) $-2.89$ rad/s and (c) $-6.83$ rad/s, each for a duration of $-0.798$ s.

Fig. 12. Graph of estimated angular velocities of elements with non-zero velocities.

velocity measurement (imaging) technique were applied in their study, there is a possibility that they could have obtained angular velocities of mixing streams at each plane. In addition, a cross-correlation of the images at adjacent planes could possibly have resulted in estimates of the axial velocities of the mixing stream.

There are some processes that inherently present dynamically perturbed resistivity fields. For such processes, EIT could be used to image angular velocities without the need for a tracer object. For example, one could use EIT to map the angular velocities of a wobbling air-core in a hydrocyclone relative to the axis of the hydrocyclone apparatus.
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