Problem 1.

Characteristic equation

\[ 1 + G_e \cdot G_v \cdot G_p \cdot G_m = 0 \]
\[ G_p = \frac{2}{3S+1}, \quad G_m = 2, \quad G_v = 0.25 \]
\[ G_e = K_e \cdot \left( 1 + \frac{1}{10S} \right) \]
\[ 1 + K_e \cdot \left( 1 + \frac{1}{10S} \right) \times 0.25 \times \frac{2}{3S+1} \times 2 = 1 + K_e \cdot \left( \frac{(10S+1)}{10S} \right) \cdot \left( \frac{1}{3S+1} \right) = 0 \]
\[ \Rightarrow 10S(3S+1) + K_e(10S+1) = 30S^2 + 10(1+K_e)S + K_e = 0. \]

So, the characteristic equation of this system is the following:

\[ 30S^2 + 10(1+K_e)S + K_e = 0 \quad \text{eqn (1)} \]

For a feedback controller, the open-loop gain should be greater than zero.
\[ K_c = K_e \cdot K_v \cdot K_p \cdot K_m > 0, \quad K_p = 2, \quad K_m = 2, \quad K_v = 0.25 \]
\[ \text{So,} \quad K_c > 0. \]

The roots of the equation (1) are negative at any positive \( K_c \).
So, this system is always stable as long as \( K_c \) is positive.

Routh Array:

\[
\begin{array}{c|c}
1 & K_c \\
30 & K_c \\
(1+K_c) & 0 \\
\hline
b_1 = \frac{K_c \times 10(1+K_c)}{(10S+1)} & > 0 \quad \Rightarrow \quad K_c > 0 \\
\end{array}
\]

There is no upper-bound on \( K_c \), so the ultimate gain \( (K_u) \) cannot be determined → Increasing \( K_c \) cannot make this process unstable as long as \( K_c \) is positive.
Problem 2.

\[ G_v = 2, \ G_m = K_m = 1, \ G_d = G_p = \frac{2}{(5s+1)(0.5+1)} \]

\[(T / T_{sp})_d = \frac{1}{3s+1}, \ \text{where} \ (T / T_{sp})_d \ \text{is a desired closed-loop transfer function.} \]

The controller by using direct synthesis method:

\[
G_c = \frac{1}{G} \left[ \frac{(T / T_{sp})_d}{1 - (T / T_{sp})_d} \right] \quad \text{--- eqn (2)}
\]

where \( \bar{G} \) is an approximation of an actual process, \( G \), and

\[ G = G_v G_p G_m. \]

For this problem, let's assume that the process model is perfect (\( \bar{G} = G \)).

By substituting the given \((T / T_{sp})_d\) into equation (2) and solving for \( G_c \), the controller design equation becomes

\[
G_c = \frac{1}{G} \left[ \frac{1}{1 - \frac{1}{3s+1}} \right] = \frac{1}{G} \left[ \frac{1}{\frac{1}{3s+1}} \right] = \frac{1}{\frac{5s+1}{12s}} = \frac{12s}{5s+1} = \frac{2 \times 2 \times 1 \times 3s}{5s+1} \quad \text{--- equation (3)}
\]

The parallel form of the PID controller is

\[
\frac{P'(s)}{E(s)} = K_c \left[ 1 + \frac{1}{T_e s} + T_0 s \right] \quad \text{--- equation (4)}
\]

Rearrange equation (3)

\[
G_c = \frac{15}{12} + \frac{1}{12s} + \frac{50s}{12s} = \frac{15}{12} \left[ 1 + \frac{12}{15} \cdot \frac{1}{2s} + \frac{12 \cdot 50}{15 \cdot 12s} \right]
\]

= \frac{1}{2s} \left[ 1 + \frac{1}{15s} + \frac{1}{3.33s} \right] \quad \text{--- equation (5)}

By comparing equations (4) and (5), we can conclude the following:

\[ K_c = 1.25, \ T_e = 12, \ T_0 = 3.33 \]

From the problem statement, closed-loop transfer function is \( \frac{1}{3s+1} \), and gain of this function is 1. → Integral action is included because there is no offset.
Problem 3.

(a) The ideal proportions of natural gas and air vary directly with the BTU content of the fuel. When the heating value, which is unmeasurable disturbance, of the natural gas varies, the optimum molar flow ratio between the air and the fuel would need to be changed accordingly to maintain the temperature of the system at a desired temperature. But this cannot be done without feedback control.

(b) Energy content of natural gas is variable from 900 Btu/scf to 1100 Btu/scf and depends on its accumulations which are influenced by the amount and types of energy gases they contain such as CO₂, CO, N₂, H₂S, and etc. The composition of natural gas varies depending on the field, formation, or reservoir from which it is extracted.

(c) Use feedforward-feedback control.

   - we can measure the temperature of the system and use an additional feedback controller along with feedforward (ratio) controller to effectively control the temperature of the system to set point.

(d) ① If there is excess air, the result is unused oxygen as well as even more nitrogen to absorb heat that is carried up the stack, resulting in energy losses.

   ② The higher excess air flow rate will produce a large amount of nitrogen oxides (NOₓ), which are harmful to human health and environment.
Problem 4.

- Cascade Control.

(a) Feed temperature becomes too high.

- In the event that the feed temperature is too high, the slave controller will sense the increase in temperature and increase the signal to the coolant valve, which will increase the flow of coolant to reduce the temperature of the feed. The master controller will sense a slight increase in temperature in the reactor and will increase the set point of the slave controller, which will in turn increase the flow rate of the coolant a second time. In this case, both the slave and the master controller work together to counteract the disturbance. As a result, the disturbance is dealt with quickly, and the reactor temperature is only affected slightly.

(b) Coolant temperature becomes too high.

- In the event that coolant temperature becomes too high, the temperature of the feed exiting the heat exchanger will increase due to a reduction in heat removal rate. The slave controller will sense this and will act as above by increasing the coolant flow rate to reduce the temperature of the heat exchanger effluent. The increased temperature of the feed will increase the reactor temperature, and the master controller will alter the set point of the slave controller. Again, the master controller and slave controller work together to counteract the disturbance.
Problem 5

\[ G_p = \frac{5}{(10s+1)(2s+1)} \quad G_t = 0.8 \quad G_v = 1 \quad G_d = \frac{2}{s+1} \]

- Process with FF control

\[ Y(s) = G_d \cdot D + G_p G_v \cdot G_f \cdot G_t \cdot D \]
\[ = (G_d + G_p G_v G_f G_t) \cdot D \]

For "perfect control", we want \( Y = 0 \) even though \( D \neq 0 \).

\[ G_d + G_p G_v G_f G_t = 0 \]
\[ G_f = -\left( \frac{G_d}{G_t \cdot G_v G_p} \right) = -\left( \frac{\frac{2}{s+1}}{0.8 \cdot 5} \right) = -\frac{1}{2} \cdot \frac{(10s+1)(2s+1)}{(s+1)} \]

This controller is physically unrealizable because the numerator is a higher order polynomial in \( s \) than the denominator.

We could approximate this controller by a physically realizable one such as a lead-lag unit, where the lead time constant is the sum of the two time constants, \( T_p_1 + T_p_2 \).

\[ T_p_1 = 10 \quad T_p_2 = 2 \quad \rightarrow \quad T_p_1 + T_p_2 = 10 + 2 = 12 \]

\[ \therefore \quad G_f, modified = -\frac{1}{2} \cdot \frac{(12s+1)}{(s+1)} \]

or ignore small time constant \( T_p_2 \).

\[ \therefore \quad G_f, modified = -\frac{1}{2} \cdot \frac{(10s+1)}{(s+1)} \]