1. (40pts) A steam heated stirred tank (similar to the one discussed in the lecture notes for Chapter 2) has the following operating conditions and constant parameters:

\[ T_i = 40°C \]

\[ \Delta H_v = 600 \frac{cal}{g} \]

\[ C = 1 \frac{cal}{g°C} \]

\[ w = \frac{10^4 kg}{hr} \]

\[ \rho = \frac{10^3 kg}{m^3} \]

\[ V = 100 m^3 \]

The MV is the steam rate and the CV is the tank temperature. The set point of the CV is 150°C. Assume the inlet temperature \( T_i \) stays constant for the operations below.

(a) (5pts) What is the steady state steam flow rate at the set point temperature?

(b) (5pts) Assuming a linear dynamic model in terms of deviation variables, what are the values of the process gain and the time constant (include units)?

(c) (10pts) Suppose the steam is operating at the steady state steam flow rate calculated in (a). If the steam rate is then halved, what will be the new steady state temperature? How long does it take to reach within a 1°C of the final steady state?

(d) (20pts) How can the time to reach the value in (c) be shortened? Suppose the steam rate is set to zero; how long does it take to reach \( T=95°C \)? Will the temperature remain at 95°C once it reaches the new steady state? If not, how can you keep it there? Sketch the temperature trajectory as a function of time.
2. (20 pts) The differential equation (dynamic) model for a chemical process is as follows:

\[
\frac{d^2y}{dt^2} + 2 \frac{dy}{dt} + 3y = 2u(t)
\]

where \(u(t)\) is the single input function of time. \(y(0)\) and \(dy/dt\) (0) are both zero.

What are the functions of the time (e.g., \(e^{-t/\tau}\)) in the solution to the ODE for output \(y(t)\) for \(u(t) = be^{-2t}\). \(b\) is a constant.

Note: You do not have to find \(y(t)\) in these cases. Just determine the functions of time that will appear in \(y(t)\).

3. (15 pts) Will the solution to the ODE below reach a steady state? Will it oscillate?

\[
\frac{d^2x}{dt^2} + \frac{dx}{dt} = 4 \quad \\quad \frac{dx}{dt} (t = 0) = x(t = 0) = 0
\]

Show appropriate calculations using partial fraction expansion and Laplace transforms and/or time domain analysis.

4. (25 pts) For a stirred-tank heater, assume the transfer function between the heater input change \(u(t)\) (cal/sec) and the tank temperature change \(y(t)\) (°C) can be modeled as

\[
G(s) = \frac{5}{2s+1}
\]

(a)(7 pts) Using the final value theorem, find the final (steady-state) response \(y(t)\) for a unit rectangular pulse change in the heating rate \(U(s) = \frac{1-e^{-s}}{s}\).

(b)(7 pts) Repeat the calculation in (a) for a unit ramp \(U(s) = \frac{1}{s^2}\).

(c)(11 pts) For both cases (a) and (b), sketch the responses; can you explain your answer physically?

Is there a physical limitation on the ramping of the heating rate?