Problem 6.14) Use Stokes' law's

\[ U(s) \rightarrow \frac{0.8}{(0.4s+1)^2} \rightarrow \frac{5s+0.5}{2s^2 + 3s + 1} \rightarrow Y(s) \]

\[ G(s) = \frac{4e^{-0.5s}}{(0.5s+1)(0.4s+1)(2s+1)(s+1)} \]

\[ \text{new } \theta = \theta + 0.5 \times 0.4 = 0.4 \]
\[ \text{new } T = 2 + 0.5 \]

\[ F_{OPT} = \frac{4e^{-(\theta+1.5)s}}{2.5s+1} \]

For Simulink, \( \theta = 2.3 \)

See attached Simulink.
Homework # 4 Solutions

Simulink model for original function

Simulink model for FOPTD

Plot of two models

As the model shows the FOPTD model is a relatively good fit of the original model. For this purpose the FOPTD accurately reflects the actual transfer functions.
Problem 2:

a) Fit the time constant graphically assuming that the final temperature is 25 °C.

\[ Gain \times input = K_m = (25 - 79) = -54^\circ C \]

\[ \tilde{T} = T - T_{ss} = 54 \times \left(1 - e^{-\frac{t}{\tau}}\right) \]

Find \( T \) when \( \tau = 1 \)

\[ T(t) = -54 \times (1 - 0.3679) + 79 = 44.87^\circ C \]

\( T \) is at 44.87 °C at \( t = 35.33 \) seconds therefore \( \tau = 35.33 \) seconds.

b) Minimize linear regression program by only varying \( \tau \) with \( K = 1 \).

c) Minimize linear regression program by varying \( K \) and \( \tau \).

Nonlinear Regression

![Nonlinear Regression Graph]

\[ \text{d) Part a and b look very similar. This is because they are both assuming that a given change in input should equal the same change in output (K=1). They do not fit the model very well and they intersect at the point } t = 35 \text{ which is expected because that is what was used to obtain } \tau \text{ from part a. Part c resembles the model better but still does not have as drastic of an initial descent. Hypothetically the } K \text{ should be equal to one and it appears that given infinite time the temperature would reach a steady state of 25 °C.} \]

<table>
<thead>
<tr>
<th>Part</th>
<th>( K )</th>
<th>( \tau )</th>
<th>SSE</th>
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<tbody>
<tr>
<td>a</td>
<td>1</td>
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<tr>
<td>b</td>
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